An active learning-oriented error-based stopping criterion for efficient structural reliability analysis

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Background

In recent decade, the surrogate-based active learning reliability analysis methods (as shown in Fig. 1) have gained extensive attention for their capability to achieve an excellent trade-off between accuracy and efficiency.

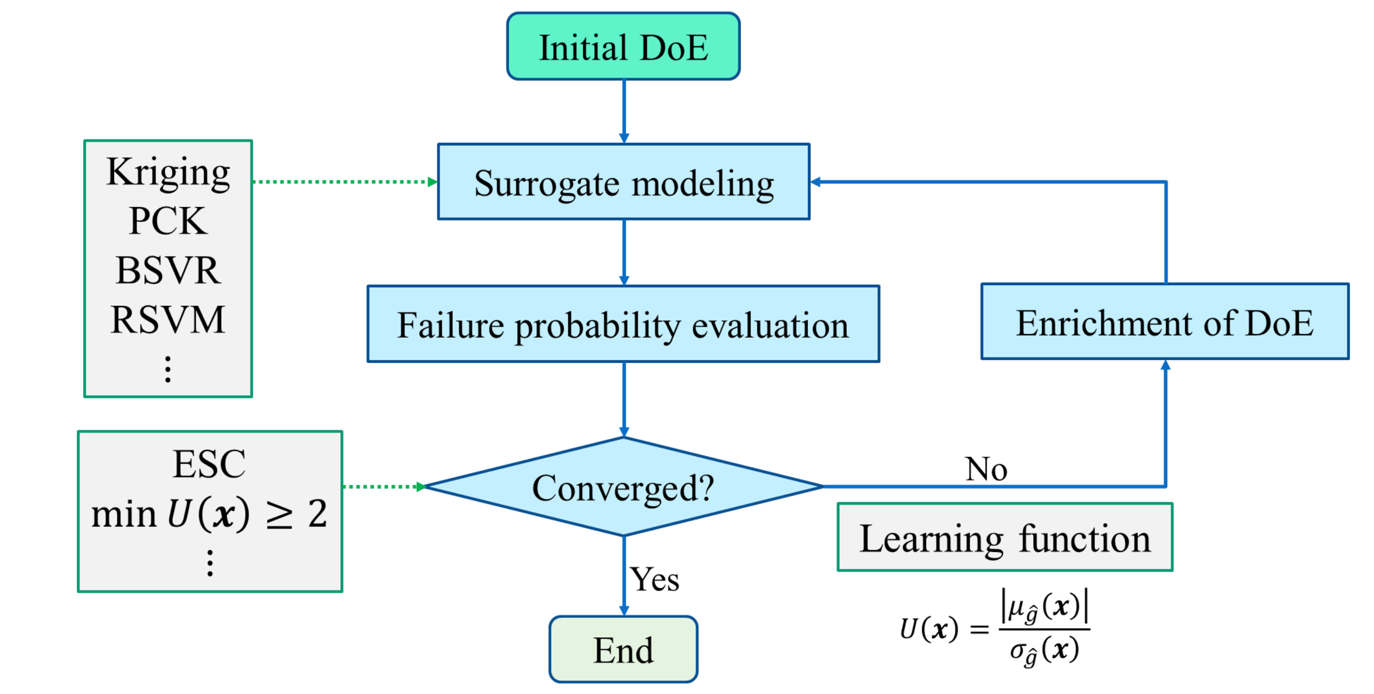


Fig. 1. The surrogate-based active learning method for reliability analysis.

The focus of this work is to develop an efficient algorithm to terminate the active learning process at an appropriate stage. Specifically, an error-based stopping criterion (ESC) is derived according to the Chebyshev's inequality, whereby the upper bound of the failure probability estimation error can be easily calculated without entailing any assumption or the bootstrap resampling analysis. Thereafter, a hybrid stopping criterion that accounts for the estimation error and its stabilization property at the converged stage is developed to enhance the computational efficiency of active learning methods. For illustration purposes, the proposed stopping criterion is integrated with the Bayesian support vector regression (BSVR) [1,2] to form the ABSVR method for efficient structural reliability analysis.

Methods

The error-based stopping criterion (ESC) was originally developed in [1], and can be expressed as:

$$\epsilon_r \leq \max\left(\left|\frac{\hat{N}_f}{\hat{N}_f-\hat{N}_{fs}^u}-1\right|,\left|\frac{\hat{N}_f}{\hat{N}_f+\hat{N}_{sf}^u}-1\right|\right)=\hat{\epsilon}_{\max} \leq \epsilon_{tol} \quad (1)$$

where $\hat{\epsilon}_{\max}$ is the upper bound of the failure probability error ϵ_r; ϵ_{tol} is the threshold value.

The key for ESC is to obtain the confidence interval of the number of samples ($\hat{N}_{fs}$ and $\hat{N}_{sf}$) whose sign is being wrongly predicted by a surrogate model. To date, the confidence interval can be derived according to the probabilistic properties of Poisson binomial distribution [3] or using the bootstrap resampling method (**BESC**) [4]. These two approaches either involve assumptions that might be invalid for a small sample size or entail additional computational costs. In this study, Chebyshev's inequality is adopted to derive the confidence interval of $\widehat{N}_{fs}$ and $\hat{N}_{sf}$ (**CESC**).

$$P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2} \rightarrow \begin{cases} P\left(\mu-\frac{\sigma}{\sqrt{1-\gamma}}<X<\mu+\frac{\sigma}{\sqrt{1-\gamma}}\right)>\gamma \\ \gamma=1-\frac{1}{k^2}, \qquad 0 \leq \gamma<1 \end{cases} \quad (2)$$

Simple is beautiful!

$$\begin{cases} \hat{N}_{sf}^u=\mu_{\hat{N}_{sf}}+\frac{\sigma_{\hat{N}_{sf}}}{\sqrt{1-\gamma}} \\ \hat{N}_{fs}^u=\mu_{\hat{N}_{fs}}+\frac{\sigma_{\hat{N}_{fs}}}{\sqrt{1-\gamma}} \end{cases} \quad (3)$$

Then, a hybrid stopping criterion (**HESC**) that can exploit the stabilization property of the failure probability is proposed as follows [5]:

$$\begin{cases} \left|\frac{\hat{P}_f^i-\hat{P}_f^{i-1}}{\hat{P}_f^{i-1}}\right| \leq \epsilon_{tol1} \quad \text{and} \quad \left|\frac{\hat{P}_f^{i-1}-\hat{P}_f^{i-2}}{\hat{P}_f^{i-2}}\right| \leq \epsilon_{tol1}, i \geq 3 \\ \max\left(\left|\frac{\hat{N}_f}{\hat{N}_f-\hat{N}_{fs}^u}-1\right|,\left|\frac{\hat{N}_f}{\hat{N}_f+\hat{N}_{sf}^u}-1\right|\right)=\hat{\epsilon}_{\max} \leq \epsilon_{tol2} \end{cases} \quad (4)$$

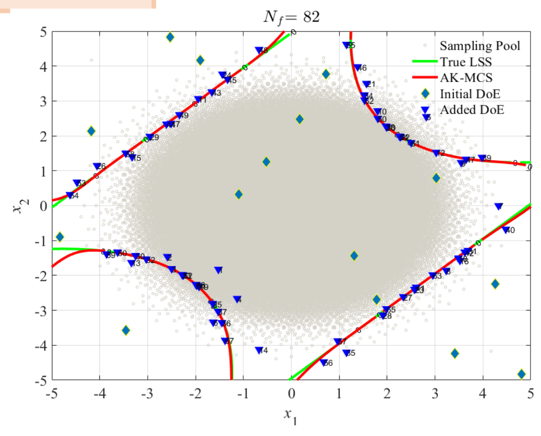
$$\max\left(\left|\frac{\hat{N}_f}{\hat{N}_f-\hat{N}_{fs}^u}-1\right|,\left|\frac{\hat{N}_f}{\hat{N}_f+\hat{N}_{sf}^u}-1\right|\right)=\hat{\epsilon}_{\max} \leq \epsilon_{tol} \quad (5)$$

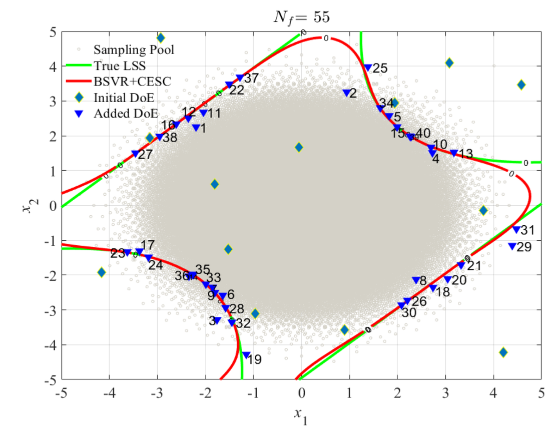
where ϵ_{tol}= 0.01, ϵ_{tol2}= 0.1, and ϵ_{tol1} = 0.001.

Numerical Examples

A series system with four branches

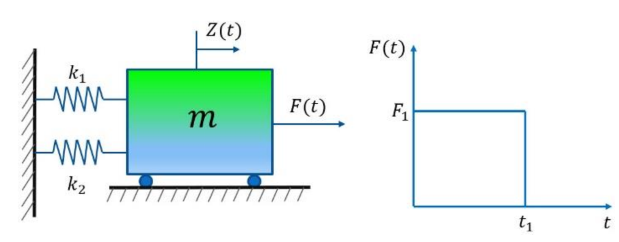
$$g(\boldsymbol{x}) = \min \begin{Bmatrix} 3+0.1(x_1-x_2)^2-\frac{x_1+x_2}{\sqrt{2}} \\ 3+0.1(x_1-x_2)^2+\frac{x_1+x_2}{\sqrt{2}} \\ (x_1-x_2)+\frac{7}{\sqrt{2}} \\ (x_2-x_1)+\frac{7}{\sqrt{2}} \end{Bmatrix}$$





Methods	$\hat{P}_f$	$\hat{\beta}$	N_f	$\epsilon_{\hat{P}_f}(\%)$
MCS	2.221×10^{-3}	2.845	1×10^7	–
AK-MCS	2.235×10^{-3}	2.843	84.9	0.63
ESC+U	2.229×10^{-3}	2.844	57.8	0.36
BESC+U	2.215×10^{-3}	2.846	57.5	0.27
CESC+U	2.226×10^{-3}	2.844	58.6	0.23
HESC+U	2.214×10^{-3}	2.845	55.2	0.32

A nonlinear oscillator

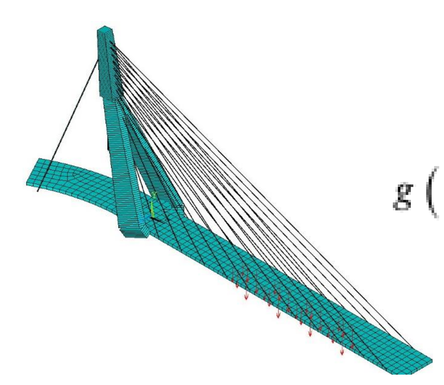


Random variable	Distribution	Mean	Standard deviation
m	Normal	1	0.05
c_1	Normal	1	0.1
c_2	Normal	0.1	0.01
r	Normal	0.5	0.05
t_1	Normal	1	0.2
F_1	Normal	1	0.2

$$g(c_1, c_2, m, r, t_1, F_1) = 3r - \left|\frac{2F_1}{m\omega_0^2}\sin\left(\frac{\omega_0 t_1}{2}\right)\right|$$

Methods	$\hat{P}_f$	$\hat{\beta}$	N_f	$\epsilon_{\hat{P}_f}(\%)$
MCS	2.859×10^{-3}	1.902	1×10^7	–
AK-MCS	2.852×10^{-3}	1.903	530	0.24
ESC+U	2.863×10^{-3}	1.902	55.8	0.14
BESC+U	2.856×10^{-3}	1.902	52.7	0.10
CESC+U	2.861×10^{-3}	1.901	54.5	0.07
HESC+U	2.854×10^{-3}	1.902	46.9	0.17

A single tower cable-stayed bridge



$$g(E_1, E_2, D_1, D_2, F_1, F_2, F_3) = \Delta_{limit} - |\Delta_{\max}|$$

Methods	$\hat{P}_f$	$\hat{\beta}$	N_f	$\epsilon_{\hat{P}_f}(\%)$
MCS	6.732×10^{-2}	1.496	1×10^5	–
AK-MCS	6.804×10^{-2}	1.491	190.8	1.07
ESC+U	6.791×10^{-2}	1.491	53.5	0.88
BESC+U	6.663×10^{-2}	1.500	55.5	1.02
CESC+U	6.765×10^{-2}	1.490	54.0	0.49
HESC+U	6.800×10^{-2}	1.489	40.3	1.01

Conclusions

1. The ESC derived from Chebyshev's inequality (CESC) can provide results with comparable accuracy and efficiency as its counterparts, i.e., ESC and BESC.
2. The proposed hybrid stopping criterion (HESC) is generally more efficient than its ESC-based counterparts, without compromising the estimation accuracy.
3. The proposed criterion is simple and entails no assumptions nor bootstrap analysis; can be integrated with any surrogate-based method for efficient reliability analysis.

References

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